NASA TECHNICAL TRANSLATION

NASA TT F-14,918

THE KUTTA-JOUKOWSKY CONDITION IN THREE-DIMENSIONAL FLOW

Robert Legendre

(NASA-TT-F-14918) THE KUTTA-JOUKOWSKY CONDITIONS IN THREE-DIMENSIONAL FLOW (Linguistic Systems, Inc., Cambridge, Mass.) 24 p HC $3.25

N73-24319

Unclas

H3/12 04670

Translation of: "La condition de Joukowski en ecoulement tridimensionnel," La Recherche Aerospatiale, 1972, no. 5 (Sept.-Oct.), pp. 241-247.

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D.C. 20546 MAY 1973

STANDARD TITLE PAGE

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TT F-14,918		
4. Title and Subtitle: THE KUTTA-JOUKOWSKY CONDITION IN THREE-DIMENSIONAL FLOW		5. Report Date: MAY 1973
		6. Performing Organization Code
7. Author(s): Robert Legendre		8. Performing Organization Report No.
		10. Work Unit No.
9. Performing Organization Name and Address: LINGUISTIC SYSTEMS, INC. 116 AUSTIN STREET CAMBRIDGE, MASSACHUSETTS 02139		11. Contract or Grant No.: NASW-2482
		13. Type of Report & Period Covered: TRANSLATION
12. Sponsoring Agency Name and Address: NATIONAL AERONAUTICS AND SPACE ADMINISTRATION WASHINGTON, D.C. 20546		14. Sponsoring Agency Code

15. Supplementary Notes

Translation of: "La condition de Joukowski en ecoulement tridimensionnel," La Recherche Aerospatiale, 1972, no. 5 (Sept.-Oct.), pp. 241-247.

16. Abstract

The separation line along which a vortex sheet is attached on a wing is not always limited to the conventional trailing edge. It may extend to wing tips or even to parts of the leading edges.

From observations of the flow over marine propeller models and delta wing models, a discussion is started, aiming at improving the description of the flow over any wing, and giving a better basis for an accurate calculation of the perfect fluid flow used as a reference.

17. Key Words (Selected by Author(s))	18. Distribution Statement
	UNCLASSIFIED - UNLIMITED

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price
UNCLASSIFIED	UNCLASSIFIED	22	3.25

NASA TT F-14,918

THE KUTTA-JOUKOWSKY CONDITION IN THREE-DIMENSIONAL FLOW†

Robert Legendre††

1. Introduction

The supporting surface theory of Ludwig Prandtl and particularly his diagramming of the so-called supporting line have enabled predictions to be made on the lift of elongated wings and even distribution of this lift along the wingspan, with sufficient accuracy for sizing the structure. /242*

With the increase in aircraft speed, wings have been decreased in length and swept further back. The supporting surface theory has been substituted for the supporting line concept.

Calculation of the exact shape of the vortex sheet could be passed over, since modifications to this shape have only a minor influence on the distribution of pressure on the wing. However, control experiments have revealed anomalies, even for relatively small incidences, particularly at the wing tips.

It is too easily accepted that experimental checking cannot fully take into account the predictions established for a flow pattern in ideal fluid, and that it is thus futile to keep on perfecting such a pattern.

In quite a number of cases, certain patterns of behavior in real fluid,apparently abnormal and attributed in too facile a manner to viscosity effects,have been explained by the ideal fluid calculation.

---

†Communication presented at the 13th Congress of I.U.T.A.M., Moscow, August, 1972.

††Member of the Academy of Sciences, Senior Scientific Adviser to O.N.E.R.A.

*Numbers in righthand margin indicate pagination of foreign text.

Since the advent of powerful computers, it has become possible to determine the shape of the vortex sheet over several mean chords of the wing and establish curvature of the sheet's edges as a fully-predictable phenomenon. Two-dimensional calculations in sections distant from the wake took this curvature into account only very incompletely. We still have to determine exactly the length of the line of the wing from which the sheet has originated.

The present article is devoted to discussion of this problem.

2. Marine Propellers

The blades of marine propellers for fast ships are short supporting surfaces, which were very rapidly introduced into naval engineering.

The appearance of cavitation must be delayed, or its development tempered, by giving a large surface to the blades, but their diameter is limited since the propellers must not extend too far beyond the ship's beam.

In 1932, while a young engineer at the Carenes, Paris Test Tank, the author was taken to participate in propeller studies for the liner "Normandie". I first familiarized myself with the theory, far ahead of its time, developed by my friend Roger Brard, now Chairman of the Paris Academy of Sciences, then interpreted the observations made in the viewing channel on flow around the propeller models.

At the time, flow was made visible in the water by suspending fine particles of aluminum by a process largely developed by Camichel at Toulouse. Moreover, certain propeller models were arranged for distribution of air bubbles at various points suitably chosen on the surface of the blades. Stereoscopic photographs taken with adjustable exposure times enabled the three-dimensional nature of the flow to be restored fairly correctly and, in particular, the curvature of the vortex sheets, as shown by the heliocoidal trajectories of the air bubbles, to be clearly observed.

It appeared to the author that the origin of curvature was not the point where the blade was tangential to the maximum cylinder of the propeller but at a point which is usually considered as the leading edge, located far upstream, where the angle of the tangent to this pseudo-leading edge with the radius is about 45° (Fig. 1).

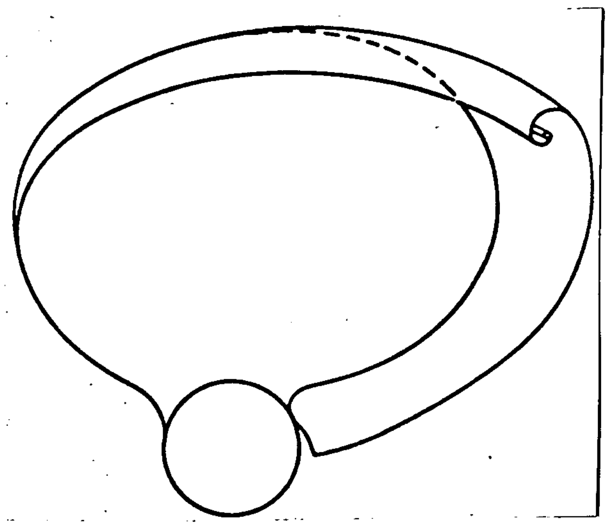

Fig. 1. Formation of a vortex sheet on a marine propeller blade.

These observations were discussed at great length between the author and his supervisor, Emile Barrillon. Tests were conducted at very low Reynolds numbers, and viscosity could be held responsible. However, this too-simple hypothesis could not be retained, and already the idea that the Joukowsky condition could apply to a substantial fraction of the leading edge with very low viscosity was formulated.

It was out of the question to work out this idea at the time. The limited computing facilities available were already overflowing by processing Roger Brard's theory, and it was impossible to begin a truly three-dimensional flow calculation. However, it was important for the future to have cast doubt on the validity of a perfectly clear boundary between the leading edge and trailing edge, and to have suggested for the first time that Joukowsky's condition could apply to a leading edge under certain conditions.

It is possible to make the idea intelligible in the vernacular by stating that the fluid cannot "remember" the orientation of its infinite speed upstream to reconstitute in addition, in the case of a propeller, the transition velocity triangle from the fixed reference point to the moving reference point. Any surface line to which the fluid is moving, on both sides of this line, is a trailing edge independently of any geometrical consideration that is too elementary.

Unable to make calculations, Emile Barrillon asked several of his colleagues in the Academy whether the problem of determining the flow of the ideal fluid would be properly formulated if the origin of the vortex sheet was not defined from the beginning. To the knowledge of the author he received no reply and the studies he made himself gave no satisfaction. Henri Villat did take the trouble to come and discuss, with his colleague and the present author, the observations made and the theory that might be developed to take account of them.

3. Delta Wings /243

The studies undertaken in 1950 at O.N.E.R.A. directed by Maurice Roy [1] to study the behavior of delta or swallowtail wings made the above considerations once more topical.

The experiments in water with air bubbles and in the air with wires attached to the nodes of a grating, recommended by the author, and the experiments done in water by H. Werle with colored meshes, converged towards the diagram worked out by Maurice Roy of "cone-shaped vortex sheets" (Fig. 2) which he would later interpret by formulating the mathematical properties of the limiting case of the delta wing, plane, and undefined.

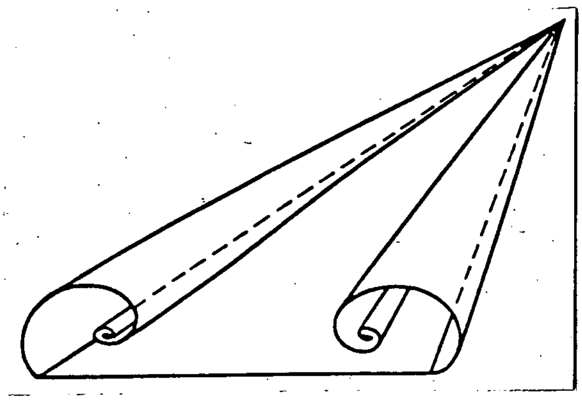

Fig. 2. Cone-shaped sheets on a delta wing.

For this diagram, corresponding to evanescent viscosity, the vortex sheet separates from the whole contour of the wing, leaving no distinction between trailing edge and leading edge.

It remained to be verified whether such a diagram fitted the properly stated mathematical problem, but the opportunities for discussion and calculation were still slim.

The author was satisfied, as were all his successors in the next two decades, with the thin-bodies theory and the conical-flow theory [2, 12]. There was no difficulty in extrapolating the law of pressure continuity through a sheet and the law of drag of the sheet by the fluid, already classical for Prandtl's sheets. The only novelties were extending the origin of the sheet to the two leading edges and writing in a Joukowsky condition on these leading edges. It remained to be verified, at the end of the calculation, that the parietal flow moved towards the leading edge as well as towards the trailing edge on both the upper and lower surfaces of the wing.

To write the correct conditions is not, however, to resolve the problem, and the author, anxious to find a digital approximation quickly, set up a simplified diagram, concentrating the entire vortex intensity on two lines, without even trying to construct a physically acceptable picture [2]. The pressure balance across the sheet was represented by the uniformity of the potential derivatives of the simplified field and the evolution of the sheet, reduced, for the angular plane wing, to the effect of a change of scale of transversal sections with increasing abscissae, had for a homolog the speed induced by the entire field in the center of each vortex. Because of a confusion in determining a logarithm, which was corrected later, the pressure continuity across the sheet was not satisfied in the first calculation.

Later, Brown and Michael in the United States [4], Mangler and Smith in Great Britain [9], and Gorston in Germany [10] took the work of O.N.E.R.A. and simplified less grossly than had the author the conical sheets described and interpreted by Maurice Roy. The results of Mangler and Smith are particularly accurate and give a good determination of the shape of the cone sheet.

Today, it is possible to study not only the permanent flow but the nonstationary flow around a moving wing or a wing subject to vibration. It is not necessary to assume that the leading edges are rectilinear and that the wing is flat, provided the sweepback angle at each point is sufficient for the calculation to supply a flow moving towards the leading edge, on both the upper and lower surfaces of the wing. The use of the thin-body theory is not essential, and the calculation may be done in the framework of three-dimensional flows or incompressible fluids, or those comparable to such fluids by the approximation of Prandtl-Glauert. It is not yet possible to calculate transsonic on supersonic flows, but it is almost certain that the major effects of vortex sheets have been worked out by the flow calculation on the incompressible fluid, and that the effects of compressibility in shocks with mechanisms independent of those of vortex sheets must be sought for. Moreover, from the point of view adopted in the present article, the laws governing the appearance of vortex sheets remain valid at transsonic or supersonic speeds, even if it is not yet possible to make exact and accurate calculation of the entire flow.

The calculations above demand very powerful computers, and the number of applications made, especially by Rehbach at O.N.E.R.A. [17, 18], remains small. It is no less important to discuss carefully the laws of formation of sheets to avoid engaging in calculations that are both costly and futile.

Rehbach formulated computer programs inspired by the work of Belotserkovskii, Butter, and Hancock [13, 16].

4. Thin Wings

It is evident that the extension to a leading edge of a Joukowsky condition, of finite speed, brings up difficulties for a wing of practical interest whose leading edge must be neither angular nor tapered. From this viewpoint, the work at O.N.E.R.A. has been wrongly interpreted by those who recommend beginning on vortex sheets at the leading edges. The directives given by Maurice Roy to his co-workers were quite clear:

- find wing shapes "adapted" to cruising speed, i.e., such that the lower and upper wing surface flows would separate on a line close to the leading edge, which strictly excludes separation of vortex sheets.

- in addition, exploit the remarkable stability of the conical sheets which appear at higher incidences so that performances evolve continuously and faithfully in a broad domain.

There was no question of confusing these two points of view by making the formation of sheets a pre-condition, even a cruising speed, since it is well known that creation of a vorticity is not gratuitous.

To properly separate the two distinct objectives without prematurely complicating them by thickness effects, it is convenient to begin by studying infinitely thin wings considering them to be thin but having a leading edge around which the flow might pass.

Development of computer techniques gave a practical answer /244 to the problem discussed between the author and Emile Barrillon: the boundary between the leading edge region, effectively separating the flow, and the region where a vortex sheet appeared is not established by computation; it is very largely arbitrary and the problem is difficult to state in mathematical terms. Other criteria must be introduced, as empirical as Joukowsky's condition itself, to define the solution to be reconciled with the experimental results.

The most striking illustration of the above statement is supplied by actual calculation of several types of flow around a highly-sweptback wing, as done by Rehbach at O.N.E.R.A. [17].

An initial solution is obtained for a vortex sheet coming only from along the conventional trailing edge (Fig. 3a). In this case, the flow passes around the leading edge and the wing tips, in theory at infinite speed: this gives no cause for concern, since it is manifested only if the leading edge is rounded and if the tip fairings had a large enough radius of curvature.

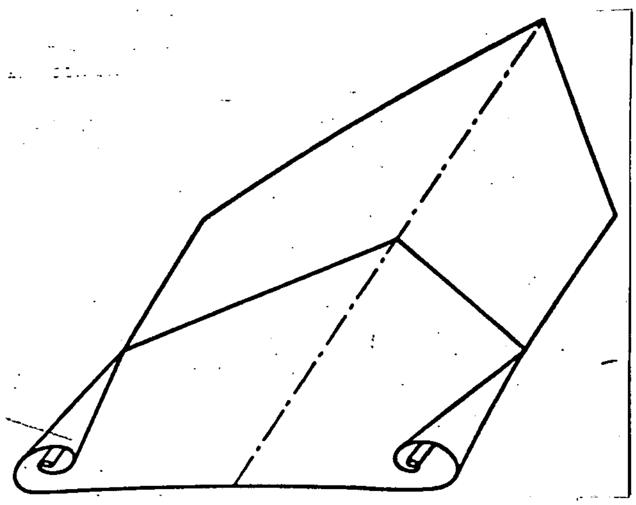

Fig. 3a. Vortex sheet coming from a trailing edge.

A second solution is obtained for a vortex sheet arising simultaneously on the conventional trailing edge and along the wing tips (Fig. 3b). The flow still passes around the leading edge.

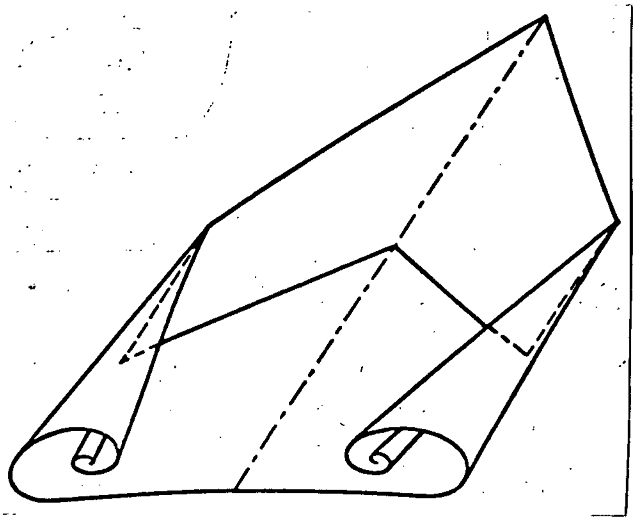

Fig. 3b. Vortex sheet coming from one trailing edge and the wing tips.

Although the computation has not yet been made it is evident that there is no fundamental difference between the wing with a very sharp sweepback angle and the delta wing, especially in the vicinity of the apex. It is thus possible to find a solution such that the vortex sheet is emitted over the entire contour of the wing (Fig. 3c).

Finally, we can look at the possibility of selecting the origin of the sheet at an arbitrary point on the leading edge, even deciding that the sheet will pass around certain portions of the leading edge while other portions will give rise to isolated sheets. This last form of computation could provide an interpretation for the shredded sheets described by Maurice Roy from experimental results on medium swept wings [7].

The digital computations discussed above make it futile to seek to demonstrate a theorem of existence and uniqueness, at least as long as precise criteria have not been worked out.

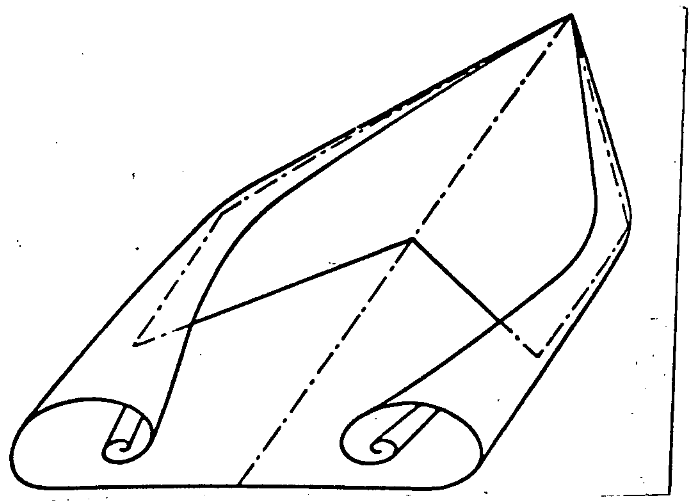

Fig. 3c. Vortex sheet coming from the whole contour of the wing.

The above comments relate to flow around a given wing. For the converse problem of designing a wing for flow to satisfy given conditions, and in particular for the first problem of adaptation raised by Maurice Roy, it is possible to have camber at the leading edge to obtain the appearance of a sheet as desired. The adaptation must be made such that a sheet would not form, and the flow would separate at the leading edge instead of converging towards the leading edge. The configuration is then two-directional. However, as the shape of the wing has been designed to satisfy these conditions, the slightest change in incidence causes the alternative of sheet separation to reappear.

To the aerodynamicist, who does not have to be concerned with properly stated problems, the freedom of interpretation he is offered is an advantage, since it allows him to adapt the ideal fluid flow picture to experimental findings and thus to search for criteria defining the solution.

5. Sharp Edges

There is little point in discussing the contouring of sharp edges since, for supporting surfaces of practical interest, such edges are placed only along the actual trailing edges onto which converge two boundary layers of viscid fluid, tending to become mixed together in a vortex sheet with evanescent viscosity. /245

Mangler and Smith [14] studied the behavior of the sheet coming from a sharp edge, and their findings are summarized below.

The basic hypothesis is that two alternatives only can arise:

- either the flow passes around the edge and the speed becomes theoretically infinite in incompressible non-cavitating fluid,
- or a vortex sheet is detached from the trailing edge, thus preventing infinite speed from being reached.

We must not attempt to substantiate this hypothesis mathematically, as it is built into the concept of ideal flow for evanescent viscosity. Only a computation including viscosity and turbulence could furnish such a substantiation, and this will be beyond our reach for some time to come.

To study the behavior of a trailing edge sheet, it is sufficient to examine the consequences of limitation of speed to a finite value.

At a point M on the sharp edge (Fig. 4) the two determinations of speed, V_e and V_i to either side of the sheet, upper and lower wing surfaces, are in the plane tangential to the sheet even if, being confused in direction, they no longer make such a tangential plane.

Moreover, the speed V_e is contained in the plane tangent to the upper surface at M, and V_i is contained in the plane tangential to the lower surface at the same point.

Thus, one of the two speeds, V_e or V_i, is tangential to the edge and the vortex sheet is tangential to the wing surface corresponding to the other speed, except perhaps in the degenerate case where both speeds are tangential to the edge or in the even more degenerate case when they are zero.

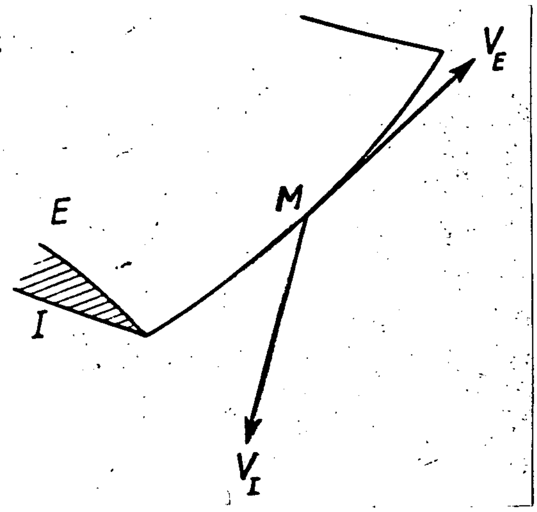

Fig. 4. Speeds at one point of an edge.

The line of reasoning assumed that the two speeds are finite but introduced no hypothesis as to their intensities. It thus remains valid if the wing is moving or subject to vibration.

If the flow is permanent, the pressure continuity across the sheet requires that the intensities of both speeds be equal, but this adds nothing essential to the conclusions reached.

However, we will examine the degenerate case in which both speeds are tangential to the edge only when the flow is permanent, since it does not appear urgent to perfect the calculation by introducing refinements encumbering the programs.

If the two speeds V_e and V_i are equal and tangential to the edge, two alternatives may be considered:

- either the two speeds are the same and circulation is stationary; for any arbitrary wing the points where this circumstance arises are isolated and, for a wing of practical interest, attacked without skidding, only the point on the longitudinal plane of symmetry satisfies the condition. It would obviously be possible to design a wing for circulation to stay constant over a finite expanse of the trailing edge, but in this case no element of the vortex sheet would have its origin on this expanse and the sheet would be torn;

- or the speeds V_e and V_i are opposed, in which case it is more difficult to show that this circumstance can arise only at isolated points. Here, it is sufficient to indicate that, if the phenomenon took place on a finite expanse of a sharp trailing edge, the parietal streamlines would travel toward two distinct nodes; even if the nodes defined at a regular point of the surface degenerated when they came to an edge, they would cause the streamlines to converge on both sides. This is because a node on the wing surface corresponds to a dip on the vortex sheet.

Actually, to extend the calculations discussed in the preceding paragraph to thick wings with a sharp tailing edge, it is sufficient to state that the product of the speed components V_e and V_i normal to the tangent of the edge is zero. At the end of the calculation, it must also be checked whether the normal component which is not zero is oriented such that the fluid leaves the wing and enters the vortex sheet [14].

Provided the origin of the sheet has been defined elsewhere, the problem seems to be properly stated, at least for digital computation. The individual isolated points where the two speeds are tangential to the trailing edge are supplied by computation.

6. Wing Tips

Engineers performing aircraft model tests in wind tunnels have discovered experimentally the importance of wing tip fairings, giving better continuity to the variable incidence and skid characteristics, both in steady state and in a changing pattern. Indeed, cone-shaped vortex sheets are not only advantageous: a rectilinear wing tip in the longitudinal plane in normal flight can give rise to very different sheets under skid conditions according to whether the sign of the skid is positive or negative. As a result undesirable wing dropping behavior arises whose disadvantages are even greater at high speeds when interaction takes place between the fields induced by the fields and shocks.

Designing wing tip fairings with adequate mean radii can allow the flow to pass around the wing tips without separations generating bumps or short bubbles, or detachment of vortex sheets.

The difficulty of generalizing the Kutta-Joukowsky condition, already in planar flow when the trailing edge is rounded, gives us to understand that it is not possible to define precisely the limit flow for evanescent viscosity around a wing tip with a fairing. If the wing sweepback is sufficiently moderate for the sheet not to appear as far forward as the leading edge, the best design is that of Fig. 3a, if the mean radius of the fairing is large, and that of Fig. 3b if this radius is negligible. Between the two extreme cases the development pattern of the three-dimensional boundary layers must be examined to decide whether the flow becomes detached or not (Fig. 5). In the case of detachment, we must /246
examine what happens to the separation line when viscosity decreases to see whether a boundary has any chance of existing. This diagram would be valid for zero viscosity.

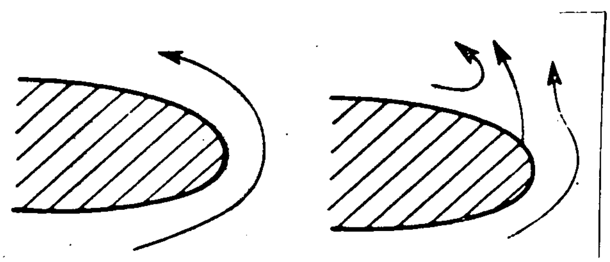

Fig. 5. Flow without and with detachment at the wing tip.

The author has always declared himself to be opposed to straight wing tips in longitudinal planes, precisely because of the ambiguity in defining an ideal reference fluid flow. He recommends tapering the wing tips leeward of the leading edge, even for substantial skid, so that they always behave just as trailing edges (Fig. 6). There is now no disadvantage to thinning them into sharp edges.

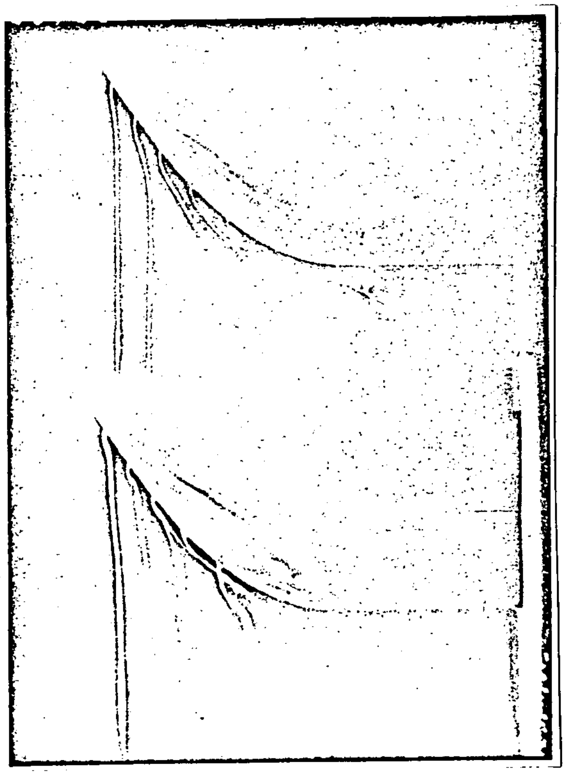

Fig. 6. Flow on the upper and lower surfaces of a wing with trailing edge tapered leeward of the trailing edge.

It is possible to avoid flow passing around the wing tips at cruising incidence when performing the adaptation calculation, but this leads to reducing local lift without reducing the friction resistance. Manufacturers will thus choose a compromise solution.

7. Leading Edge

The difficulties experienced in defining flow around the wing tips foreshadow those raised in that around the leading edge, with its failures that can lead even to complete separation, beginning formation of a cone-shaped vortex sheet.

This is why all basic research both in France and abroad has been focussed on wing models with sharp leading edges, giving edges so sharp that they behave like trailing edges. Further studies on Concorde models, whose leading edge radii were small to be compatible with supersonic speeds, confirmed the validity of such basic research, at least at high incidences. The joining of the upper and lower wing surface boundary layers, which takes place a little beyond the minimum radius of curvature, can be diagrammed by a vortex sheet in an extrapolation towards zero viscosity.

However, it would be desirable to make the separation line more precise as soon as separation is manifested so that a reference flow diagram in ideal fluid can be better defined. This would certainly be of use in the future.

Unfortunately, the research program defined by the author in collaboration with H. Werle has not thus far given significant results for two reasons: on the one hand, to a number of varying parameters, numerous models had to be built over which other work had priority, and the second, that for the only model properly tested, it appears that the Reynolds number of the viewing channel designed by Maurice Roy, although appropriate for working out the basic flow characteristics, is inadequate for detailed analysis of the origin of vortex sheets. The comments below are thus more in the nature of a priori viewpoints than solidly established experimental findings. They ask questions rather than answer them.

First of all, the radius of curvature of the leading edge and its change along the wingspan must be basic factors in separation. For the Concorde model, the local sweepback variation is also important, but seems to be less determinant than the above parameters since separation does not appear near the fuselage, although the local sweepback is considerable, because the radius of curvature is large.

The carefully-tested model is delta-shaped and, designedly, the evolution in the leading edge radius of curvature is very different from that of the Concorde. The transversal sections are elliptical and the minor axis of the ellipse is defined by a lens-shaped median longitudinal section. Thus the apex is tangential to an elliptical cone.

Unfortunately, the desire to observe the phenomena at the leading edge with a good scale led to the choice of rather a large thickness. As a result, with a very low testing Reynolds number, separation goes back to the trailing edge when incidence increases at the same time as interesting phenomena develop at the leading edge (Fig. 7).

The mechanism of vortex sheet generation is better shown on a heavily-tilted ogive which can represent a large fraction of the leading edge (Fig. 8). Colored threads show convergence of the parietal streamlines into a focus, after which they extend into the heart of the fluid with a single streamline shown up by a very fine thread. The configuration is exactly that predicted by an examination of the individual points in solutions of differential equations [5].

The phenomenon apparently has no relationship to the generation of a vortex sheet since none of the parietal streamlines either converge towards the focus nor have a particular property permitting them to be viewed as the attachment line of a sheet. The entire boundary layer of the ogive surface is evacuated in a turbulent vortex around the single thread leaving the surface.

One must imagine how the configuration described above evolves /247
when viscosity tends towards zero. The author does not believe

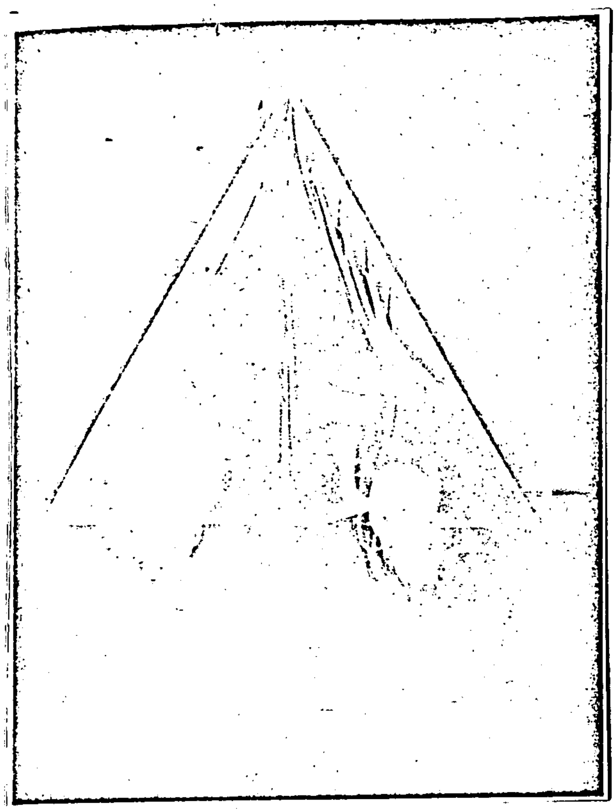

Fig. 7. Formation of foci generating vortices at the upper surface of a delta wing.

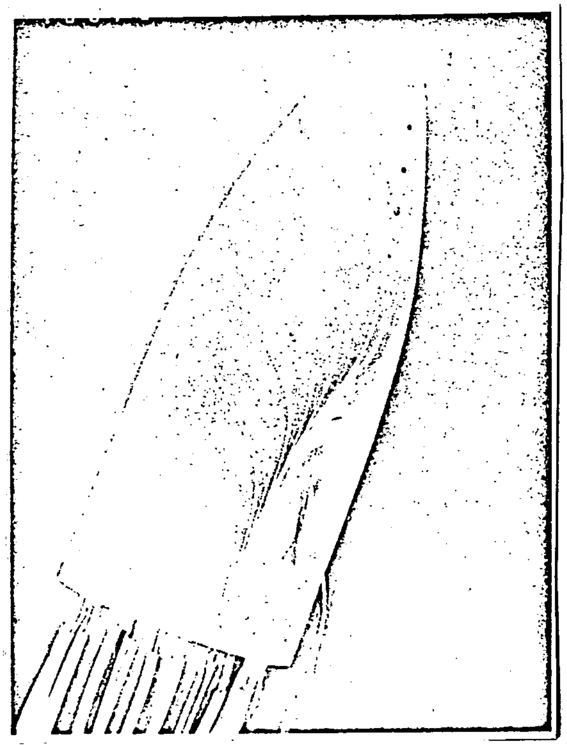

Fig. 8. Formation of a focus generating a vortex at the upper surface of a sharply tilted ogive.

that a definite answer can be deduced by logic alone. He is of the opinion that this must be given mechanically, improving the observations for smaller and smaller viscosities, and building up diagrams representing the phenomena in the best way. Just as in the case of thin wings, there is probably no single solution in an ideal fluid, but rather an infinity of solutions, all individually coherent, from which selection must be made by empirical criteria yet to be established. For example, it is possible to define a flow of ideal fluid around a delta wing with rounded leading edges by causing the vortex sheet to start from the trailing edge alone, but adding two isolated vortices normally beginning from two symmetrical points on the leading edges, which would be fairly arbitrary. The diagram derived is certainly coherent and may represent flow observed with low viscosity fairly correctly.

In addition, the comment on the quasi-isotropy of a focus is based on the hypothesis of regular behavior, which is faulty as soon as a vortex sheet appears. A diagram of ideal fluid flow with a vortex sheet based on one of these parietal streamlines ending up at the focus can thus be coherent and define a valid limit for viscosity tending towards zero.

An acceptable configuration of parietal streamlines at the upper surface of a delta wing can be reconstructed qualitatively (Fig. 9). We may imagine that the center of the focus is the origin of an isolated vortex or that the streamline leaving the throat and ending at the focus is already the start of a vortex sheet.

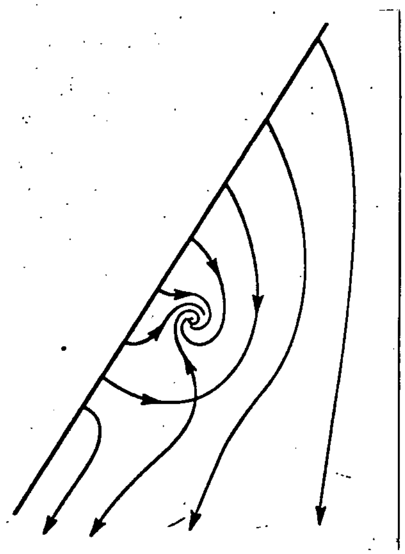

Fig. 9. Focus near the leading edge of a delta wing.

8. Conclusions

The supporting surface theory must be added to by a more precise interpretation of the anomalies appearing on wing tips, and often on large sections of the leading edge.

For this purpose, we must accept that Joukowsky's condition of finite speed on a separation line extending to the heart of the fluid by a vortex sheet at the ideal limit for the zero viscosity fluid applies not only to the traditional trailing edge defined geometrically. It can also apply to the wing tips, and even to more or less extended portions of the leading edge.

Determination of the size of the origin of sheets raises a difficult problem, the study of which is merely in its infancy, and can require the use of wind tunnels with a high Reynolds number, well-equipped with viewing facilities.

In the first stage, taking advantage of the development in /248 methods of calculating three-dimensional boundary layers, it seems possible to proceed with fairly detailed experimental research and undertake empirical studies to locate the origin of sheet formation, i.e., (as long as viscosity is not completely zero) evacuation of parietal boundary layers.

Profound understanding of the laws of separation seems to be further in the future, but its basis will certainly be closely associated with the study of nodes, throats, foci, and solutions of differential equations. The difficulty will be to predict their degeneration when viscosity tends towards zero, a difficulty mirroring those already involved in studies of degeneration when the above above special points come on single points or single points of the surface.

For the study of wings adapted to supersonic flight such as the Concorde, radii of curvature at the leading edge are rather small so that there should not be too great an uncertainty as to the location of separation lines.

It is rather on wings for low-speed subsonic flight that analysis of these phenomena is behindhand in governing the choice of suitable configurations serving as a basis for ideal fluid reference flow computation.

Manuscript submitted 8/18/72.

References

1. Roy, M., Flow characteristics around a highly swept-back wing, C. R. Ac. Sc., vol. 254, 1952, p. 2501.

2. Legendre, R., Flow in the vicinity of the forward point of a highly swept-back wing with average incidences, Rech. Aeron., no. 30 (Nov.-Dec. 1952), p. 3-8; Rech. Aeron, no. 31 (Jan.-Feb. 1953), p. 3-6, and Rech. Aeron, no. 35 (Sept.-Oct. 1953), p. 7-8.

3. Adams, M. C., Leading-edge separation from delta wings of supersonic speeds, J.A.S., vol. 20, June 1953, p. 430.

4. Brown, C. E. and W. H. Michael, Effect of leading-edge separation on the lift of a delta wing, J.A.S., vol. 21, October 1954, p. 690.

5. Legendre, R., Separation of three-dimensional laminar flow, Rech. Aeron., no. 54, Nov.-Dec. 1956, p. 3-8.

6. Roy, M., Delta wing cone sheet and apex vortex, C. R. Ac. Sc., vol. 249, no. 9, 1957, p. 1105.

7. Roy, M., Delta wing theory, Rech. Aeron, no. 56, Jan.-Feb. 1957, p. 3-12.

8. Roy, M., Notes on flow around swept-back wings, Zeit. f. ang. Math. und Phys., vol. 9 b, no. 5-6, 1958, p. 554.

9. Mangler, K. W. and J. B. Smith, A theory of the flow past a slender delta wing with leading-edge separation, Proc. Roy. Soc., vol. 251 A, no. 1265, May 1959, p. 200.

10. Gersten, K., Nonlinear supporting surface theory, expecially for airfoils with small side ratio, Ing. Arch., vol. 30, no. 6, 1961, p. 431.

11. Legendre, R., Delta wing conical vortices, C. R. Ac. Sc., vol. 257, 1963, p. 3814 and vol. 258, 1964, p. 429.

12. Legendre, R., Vortex sheets from the leading edges of delta wings, Progr. in Aeron. Sc., vol. 7, 1965.

13. Belotserkovskii, S. M., Calculation of the flow about wings of arbitrary planform at a wide range of angle of attack, Mekhanica Zhidkosti i Gaza, 1968.

14. Mangler, K. W. and J. H. B. Smith, Behaviour of the vortex sheet at the trailing-edge of a lifting wing., R. A. E. report 63.049, March 1969.

15. Legendre, R., Effect of wing tip shapes on curvature of vortex sheets, Rech. Aerosp., no. 1971, 4, p. 227-228.

16. Butter, D. J. and G. J. Hancook, A numerical method for calculating the trailing vortex system behind a swept wing at low speed, Aeron. Journal, vol. 75, August 1971.

17. Rehbach, C., Numerical study on the influence of the shape of a wing tip on curvature of the vortex sheet, Rech. Aerosp., no. 1971-6, p. 367-368.

18. Rehbach, C., Calculation of flow around thin wings with evolutional vortex sheet (to appear).